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Nonlinear Permittivity Including Non-Abelian Self-interaction of Plasmons in Quark-Gluon Plasma*

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By decomposing the distribution functions and color field to regular and fluctuation parts, the solution of the semi-classical kinetic equations of quark-gluon plasma is analyzed. Through expanding the kinetic equations of the fluctuation parts to third order, the nonlinear permittivity including the self-interaction of gauge field is obtained and a rough numerical estimate is given out for the important $\mathbf{k} = 0$ modes of the pure gluon plasma.

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During recent years, the energy changes of high energy partons traversing quark-gluon plasma(QGP) have been the subject of intensive interest [1]. To study the behavior of the high energy partons through QGP, one must constitute a kind of response theory for QGP to external current. The color permittivity, characteristic quantity of QGP media, must be known. In the frame of kinetic theory, the Abelian-like color permittivity of QGP under the linear approximation has been given out long ago [2]. Just as Ref. [3] pointed out, the self-coupling terms of color field have not been included in the calculation of relevant quantities. Ref. [4] gave out the nonlinear color permittivity of QGP beyond the linear approximation, in which the $SU(N_c)$ color algebra has been ensured in the iteration process and, to some extent, the non-Abelian characteristic of QGP has been reflected. However, there still exists the problem pointed out by Ref. [3].

In this letter, the color permittivity of QGP including the self-coupling contribution of the color field is calculated with the kinetic theory. Recently, a few works have been done by separating the distribution functions and color field into regular and fluctuation parts, with which the semi-classical kinetic equations are converted to the forms which can be solved conveniently [3–5]. The quantities in the kinetic equations of QGP, such as the quark or antiquark distribution function $f_q(x,p)$ or $f_{\bar{q}}(x,p)$, gluon distribution function $G(x,p)$, field $A(x)$ and induced color current j in QGP, can be decomposed as

$$f_q = f_q^R + f_q^T, \quad A_\mu = A_\mu^R + A_\mu^T, \quad j = j^T + j^R, \tag{1}$$

where the index R represents the regular parts of corresponding quantities, i.e., $f^R = \langle f \rangle$, with index T the stochastic fluctuation parts. $\langle \cdots \rangle$ represents taking average over a statistical ensemble. Still further, the distribution functions f_q^T,..., can be expanded as the series of small fluctuation A^T, for example

$$f_q^T = \sum_{n=1}^{\infty} f_q^{T(n)}. \tag{2}$$

With this kind of decomposition, one can obtain the kinetic equations for the fluctuation parts of quark, antiquark and gluon distribution functions from the semi-classical kinetic equations of QGP, respectively [2]. If considering only the fluctuations around the equilibrium

state, $f_q^R(f_{\bar{q}}^R)$, G^R can be chosen as Fermi-Dirac and Bose-Einstein equilibrium distribution functions, respectively. One can set $A^R = 0$ and need only to consider the evolution equation of each fluctuation quantity. For example, the fluctuation part of quark distribution function satisfies

$$
\begin{aligned}
&p^\mu \partial_\mu f_q^T = \\
&\quad ig([A_\mu^T, f_q^T] - \langle [A_\mu^T, f_q^T] \rangle) - \frac{1}{2} g p^\mu \{F_{\mu\nu L}^T, \frac{\partial f_q^R}{\partial p_\nu}\} + \frac{1}{2} i g^2 p^\mu \{[A_\mu^T, A_\nu^T] - \langle [A_\mu^T, A_\nu^T] \rangle, \frac{\partial f_q^R}{\partial p_\nu}\} \\
&\quad -\frac{1}{2} g p^\mu (\{F_{\mu\nu L}^T, \frac{\partial f_q^T}{\partial p_\nu}\} - \langle \{F_{\mu\nu L}^T, \frac{\partial f_q^T}{\partial p_\nu}\} \rangle) + \frac{1}{2} i g^2 p^\mu (\{[A_\mu^T, A_\nu^T], \frac{\partial f_q^T}{\partial p_\nu}\} - \langle \{[A_\mu^T, A_\nu^T], \frac{\partial f_q^T}{\partial p_\nu}\} \rangle).
\end{aligned}
\tag{3}
$$

The kinetic equations of the fluctuation parts of the antiquark and gluon distribution functions are similar to Eq.(3) except the opposite signs of the terms related to $\{..., ...\}$ for antiquarks; f, A and $F_{\mu\nu}$ are replaced by G, $\mathcal{A}$ and $\mathcal{F}_{\mu\nu}$ for gluons, respectively. $F_{\mu\nu} = F_{\mu\nu}^a t_a$, $\mathcal{F}_{\mu\nu} = F_{\mu\nu}^a T_a$, with $A = A^a t_a$ and $\mathcal{A} = A^a T_a$. The t_a and T_a are the corresponding generators of $SU(N_c)$ in fundamental and adjoint representation, respectively. In the following discussion we will only give out explicitly the relevant equations for f_q^T, but not for $f_{\bar{q}}^T$ and G^T unless specialized otherwise. In Eq.(3), the index L denotes the linear term of $F_{\mu\nu}$ with respect to A_μ^T.

The fluctuation part A^T of color field A obeys the following Yang-Mills equations

$$
\begin{aligned}
&\partial_\mu F_L^{T\mu\nu} = -j^{T\nu} + ig\partial_\mu([A^{T\mu}, A^{T\nu}] - \langle [A^{T\mu}, A^{T\nu}] \rangle) + \\
&\quad +ig([A_\mu^T, F_L^{T\mu\nu}] - \langle [A_\mu^T, F_L^{T\mu\nu}] \rangle) + g^2([A_\mu^T, [A^{T\mu}, A^{T\nu}]] - \langle [A_\mu^T, [A^{T\mu}, A^{T\nu}]] \rangle),
\end{aligned}
\tag{4}
$$

and $j^{T\nu}$ can be expanded according to Eq.(2), with

$$
j^{T(n)\nu} = g t^a \int \frac{d^3 p}{(2\pi)^3 p^0} p^\nu [tr t^a (f_q^{T(n)} - f_{\bar{q}}^{T(n)}) + Tr(T^a G^{T(n)})], \tag{5}
$$

being the n-th order color current.

It is convenient to work in the temporal axis gauge, i.e., $A_0 = 0$ and in the momentum space. The index T of field will be omitted as well, if without confusion. By substituting the n-th order distribution function $f^{(n)}(k, p), \cdots$ obtained from the n-th order kinetic equations into Eq.(5), one can obtain $j^{T(n)}(k)$. If expanding $j^T(k)$ up only to the first order current

$j^{T(1)}(k)$, one can recover the Abelian-like permittivity ε_L. It can be confirmed that the second order color current does not contribute to the nonlinear permittivity [4,6]. So, to obtain the nonlinear permittivity including the self-interaction of gauge field, one needs to analyze the third order equations directly.

The following shorthands

$$\sum_{k=k_1+k_2} = \int \delta(k-k_1-k_2)dk_1dk_2\frac{1}{(2\pi)^4}, \qquad \mathcal{N}_{eq} = \frac{1}{2}\left(f_q^{(0)}(p^0)+f_{\bar{q}}^{(0)}(p^0)\right)+N_cG^{(0)}(p^0).$$
$$\sum_{k=k_1+k_2+k_3} = \int \delta(k-k_1-k_2-k_3)dk_1dk_2dk_3\frac{1}{(2\pi)^8}, \qquad \chi^{\alpha\beta}(k,p) = (p\cdot k)g^{\alpha\beta}-p^\alpha k^\beta. \quad (6)$$

are introduced for notational simplicity. The third order kinetic equation is as following

$$\begin{aligned}
&-ip^\mu k_\mu f_q^{T(3)}(k,p) = \\
&igp^i \sum_{k=k_1+k_2}\Big[([A_i(k_1),f_q^{T(2)}(k_2,p)]-\langle[A_i(k_1),f_q^{T(2)}(k_2,p)]\rangle)+ \\
&+\frac{1}{2}ig\sum_{k=k_1+k_2}\chi^{i\lambda}(k_1,p)\frac{\partial}{\partial p^\lambda}\left(\{A_i(k_1),f_q^{T(2)}(k_2,p)\}-\langle\{A_i(k_1),f_q^{T(2)}(k_2,p)\}\rangle\right)+ \\
&+\frac{1}{2}ig^2\sum_{k=k_1+k_2+k_3}p^i\frac{\partial}{\partial p_j}\left(\{[A_i(k_1),A_j(k_2)],f_q^{T(1)}(k_3,p\}-\langle\{[A_i(k_1),A_j(k_2)],f_q^{T(1)}(k_3,p)\}\rangle\right)\Big].
\end{aligned} \quad (7)$$

Considering the soft excitation carrying momentum $k\sim gT$ and the momentum of particles $p\sim T$ in QGP [7,8], by substituting the third order distribution functions (7) into Eq.(5), and keeping only the leading order in g, one may identify after some lengthy calculation

$$\begin{aligned}
&j^{T(3)al}(k)\approx \\
&\Sigma^{(I)ijkl}_{k,k_1,k_2,k_3}(A_i^b(k_3)A_j^d(k_1)A_k^e(k_2)-A_i^b(k_3)\langle A_j^d(k_1)A_k^e(k_2)\rangle-\langle A_i^b(k_3)A_j^d(k_1)A_k^e(k_2)\rangle),
\end{aligned} \quad (8)$$

where

$$\Sigma^{(I)ijkl}_{k,k_1,k_2,k_3} = -g^4\sum_{k=k_1+k_2+k_3}f^{abc}f^{cde}\int\frac{d^3p}{(2\pi)^3p^0}\frac{p^ip^jp^kp^l}{p\cdot k+ip^0\epsilon}\frac{1}{p\cdot(k_1+k_2)+ip^0\epsilon}\frac{\omega_2\partial_p^0\mathcal{N}_{eq}}{p\cdot k_2+ip^0\epsilon}. \quad (9)$$

Now turn to the calculation of the non-Abelian permittivity from the mean field equation. By expanding $j^T(k)$ up to third order, substituting $j^{T(2)i}(k)$ and $j^{T(3)i}(k)$ leading order in g into Eq.(4) in temporal axis gauge and in momentum space, one can identify

$$
\begin{aligned}
&-\omega^2 A(k) + \mathbf{j}^{T(1)} \cdot \frac{\mathbf{k}}{K} \\
\approx & -g^2 t^a \sum_{k=k_1+k_2+k_3} \left\{ g^2 \int \frac{d^3p}{(2\pi)^3 p^0} f^{abc} f^{cde} \frac{1}{p\cdot k + ip^0\epsilon} \frac{1}{p\cdot(k_1+k_2)+ip^0\epsilon} \right. \\
& \times \frac{\mathbf{p}\cdot\mathbf{k}}{K} \frac{\mathbf{p}\cdot\mathbf{k}_1}{K_1} \frac{\mathbf{p}\cdot\mathbf{k}_2}{K_2} \frac{\mathbf{p}\cdot\mathbf{k}_3}{K_3} \frac{\omega_2 \partial_p^0 \mathcal{N}_{eq}}{p\cdot k_2 + ip^0\epsilon} \\
& \times \Big(A^b(k_3) A^d(k_1) A^e(k_2) - A^b(k_3) \langle A^d(k_1) A^e(k_2) \rangle - \langle A^b(k_3) A^d(k_1) A^e(k_2) \rangle \Big) \\
& \left. - \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{K_1 K_2} \frac{\mathbf{k}\cdot\mathbf{k}_3}{K K_3} f^{abc} f^{cde} \Big[A^b(k_3) A^d(k_1) A^e(k_2) - \langle A^b(k_3) A^d(k_1) A^e(k_2) \rangle \Big] \right\}. \qquad (10)
\end{aligned}
$$

The first term on the right-hand side of above equation with the factor in big parentheses corresponds to the interaction between the particles and the secondary waves resulting from the nonlinear interactions of the eigenwaves in QGP. It should be stressed that the second term on the right-hand side of Eq.(10) with the factor in square brackets is the plasmon self-interaction term in QGP, which is characteristic of the QCD plasma and different from the QED. It is well known that the solution of the Yang-Mills equation in QCD vacuum is very difficult because of the self-coupling term. Just as Ref. [3] pointed out that, in previous works, this term reflecting the non-Abelian characteristics has been discarded. However, in hot dense medium where the phase of the excitation of field is random, one can take average over phase in the process of calculation [9] and make it possible to solve the mean field equation (4) or (10).

Multiplying Eq.(10) with $A^g(k')$, then taking average with respect to the random phase, one can identify [9]

$$
\left(\varepsilon^L \delta_{ad} + \varepsilon^{NL}_{ad} \right) \langle dg \rangle = 0, \qquad (11)
$$

with the notation $\langle dg \rangle = \langle A^d A^g \rangle_k$ representing the correlation strength of color field. The nonlinear permittivity defined in Eq.(11) depends on the correlation strength

$$
\begin{aligned}
\varepsilon^{NL}_{ad} &\equiv \varepsilon^S_{ad} + \varepsilon^R_{ad}, \qquad (12) \\
\varepsilon^S_{ad} &= -\frac{g^2}{\omega^2} \int \left[f^{abc} f^{cfd} \langle bf \rangle_{k_1} + \left(\frac{\mathbf{k}\cdot\mathbf{k}_1}{K K_1} \right)^2 \left(f^{abc} f^{cde} \langle be \rangle_{k_1} + f^{adc} f^{cfe} \langle fe \rangle_{k_1} \right) \right] \frac{dk_1}{(2\pi)^4}, \\
\varepsilon^R_{ad} &= -\frac{g^4}{\omega^2} \int \frac{d^3p \partial_p^0 \mathcal{N}_{eq}}{(2\pi)^3 p^0} f^{abc} \left(\frac{\mathbf{p}\cdot\mathbf{k}_1}{K_1} \right)^2 \left(\frac{\mathbf{p}\cdot\mathbf{k}}{K} \right)^2 \frac{1}{p\cdot k + ip^0\epsilon} \\
& \quad \times \frac{1}{p\cdot(k-k_1)+ip^0\epsilon} \left(\frac{\omega}{p\cdot k + ip^0\epsilon} f^{cfd} \langle bf \rangle_{k_1} + \frac{\omega_1}{p\cdot k_1 + ip^0\epsilon} f^{cde} \langle be \rangle_{k_1} \right) \frac{dk_1}{(2\pi)^4}.
\end{aligned}
$$

The ε^S_{ad} comes from the self-interaction of plasmons, while the ε^R_{ad} represents the contribution from the the interactions between the plasma particles and the beating of two timelike eigenwaves. The factors $(p\cdot k+ip^0\epsilon)^{-1}$, $(p\cdot k_1+ip^0\epsilon)^{-1}$, $(p\cdot(k-k_1)+ip^0\epsilon)^{-1}$ in ε^R_{ad} have different contributions to the integrals. The two former factors have only real contributions because the eigenwaves in QGP are always timelike [2], while the latter has both real and imaginary contributions to ε^R_{ad} because the beating of two timelike eigenwaves can be spacelike by noticing the crucial relation [10]

$$\frac{1}{p\cdot(k-k_1)+ip^0\epsilon}=P\frac{1}{p\cdot(k-k_1)}-i\frac{\pi}{p^0}\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}_1}-\frac{\mathbf{p}}{p^0}\cdot(\mathbf{k}-\mathbf{k}_1)). \tag{13}$$

From Eq.(12), one can conclude that the nonlinear permittivity ε^{NL}_{ad} is a matrix in color space and depends on the correlation $\langle A^bA^c\rangle_k$. A rough value estimate can make us see the contributions of ε^S_{ad} and ε^R_{ad} to the color permittivity more clearly. One can easily calculate the diagonal elements that is considering only the correlation of the same color by using

$$\langle bc\rangle_k=-\frac{\pi}{\omega^2}(\delta(\omega-\omega_{\mathbf{k}})+\delta(\omega+\omega_{\mathbf{k}}))I_{\mathbf{k}}\delta_{bc}, \tag{14}$$

where $I_{\mathbf{k}}$ characterizes the total intensity of the fluctuating oscillation with frequency $\omega_{\mathbf{k}}$. Furthermore, if considering the fluctuation of thermal level, i.e., one can take $I_{\mathbf{k}}=4\pi T$. By taking the upper limit of K_1 to be gT while the lower limit g^2T, the real part of the diagonal elements of ε^{NL}_{ad} can be easily obtained for the important $\mathbf{k}=0$ modes of the pure gluon plasma occasion

$$Re(\varepsilon^{NL}_{ad})=\delta^{ad}(\varepsilon^S+Re\varepsilon^R)\approx-\delta^{ad}(0.976g+1.913g). \tag{15}$$

Summarizing, the non-Abelian permittivity (12) including the self-interaction of gauge field is given out for the first time. It relies on the concrete modes $\omega_{\mathbf{k}}$ and the correlation of field as in the electro-magnetic plasma. However, It is truly non-Abelian, because of the presences of color indices and the $SU(N_c)$ structure constants. The motivations for us to calculate the permittivity including the self-interaction are the following: **1**. The gauge invariance is a embarrassing and important problem. Without considering the self-interaction, the $SU(N_c)$ gauge symmetry will be violated. The color permittivity reflecting the characteristic of QGP must include the influence of self-interaction. **2**.The non-Abelian permittivity

contributed by the self-interaction will influence the relevant physical quantities of QGP. For example, the rough numerical result (15) indicates that the nonlinear eigenfrequency shift is changed greatly by this effect [9]

$$\Delta\omega^S = -\frac{\varepsilon^S}{\frac{\partial\varepsilon^L(\omega,\mathbf{k})}{\partial\omega}\big|_{\omega_{\mathbf{k}}}} = 0.282g^2T.$$

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